

# $P_g(k)$ near horizon scales: galaxy bias in general relativity and effective $f_{NL}$

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Theoretical **A**stro**P**hysics **I**ncluding **R**elativity, CalTech

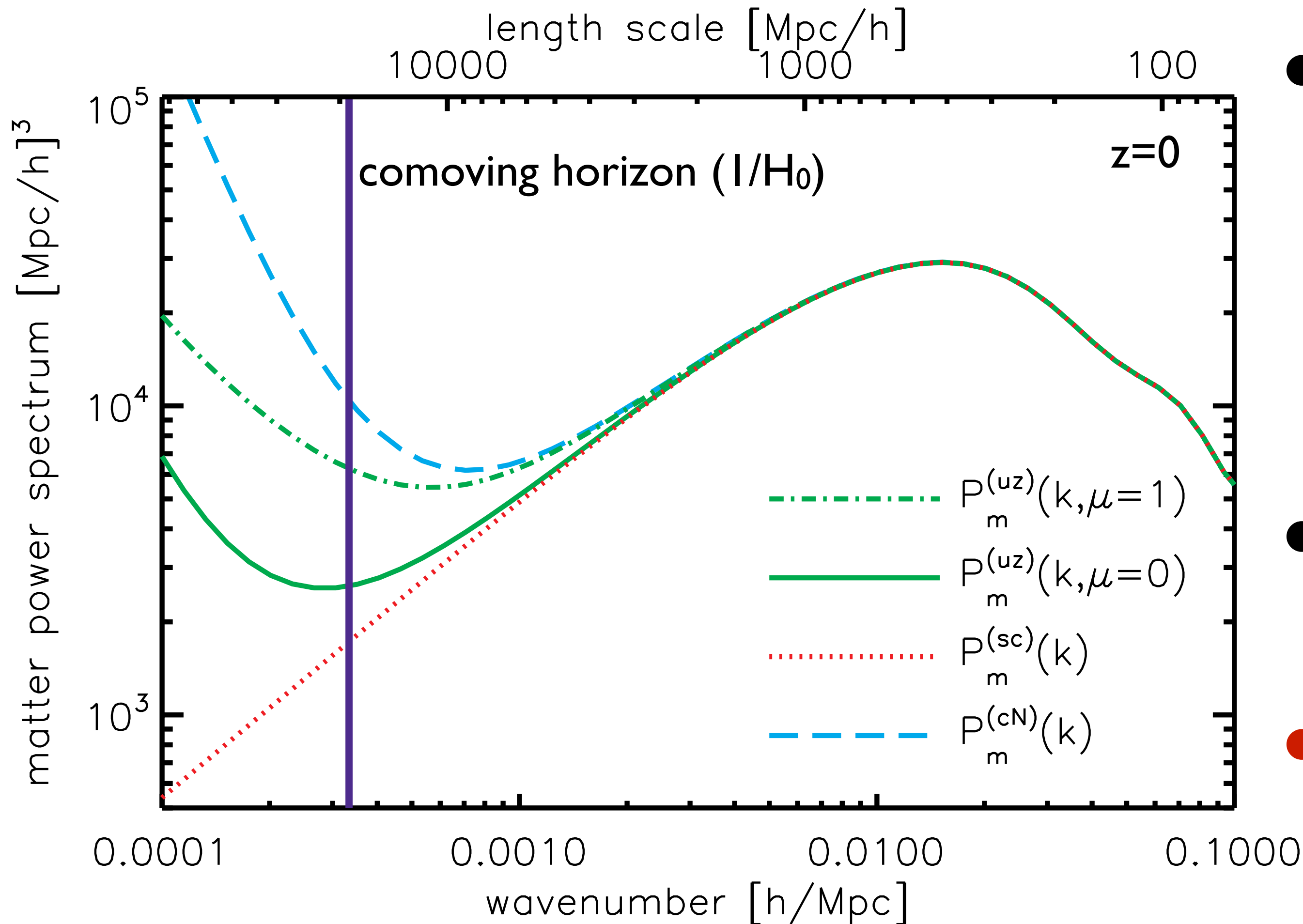
Cosmological non-Gaussianity: observations confront theory workshop

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Work in progress with Fabian Schmidt and Chris Hirata

linear bias = linear in matter density

$$P_g(k) = b^2 P_m(k). \text{ But, which } P_m(k)?$$



- $P_m(k)$  from three gauges
- conformal Newtonian (cN)
- synchronous comoving (sc)
- uniform redshift (uz)
- General covariance says all  $P(k)$ s are equally good.
- **Q: what is  $P_g(k)$  we will measure in the large scale galaxy surveys?**

# Difference in background

- Two coordinate systems (gauge)  $x_A$  and  $x_B$  [ $x=(\tau, x^i)$ ]

$$n = \bar{n}(x_A) + \delta n(x_A) = \bar{n}(x_B) + \delta n(x_B)$$

Scalar variable

- $\delta n_A$  with B coordinate variables (gauge transformation):

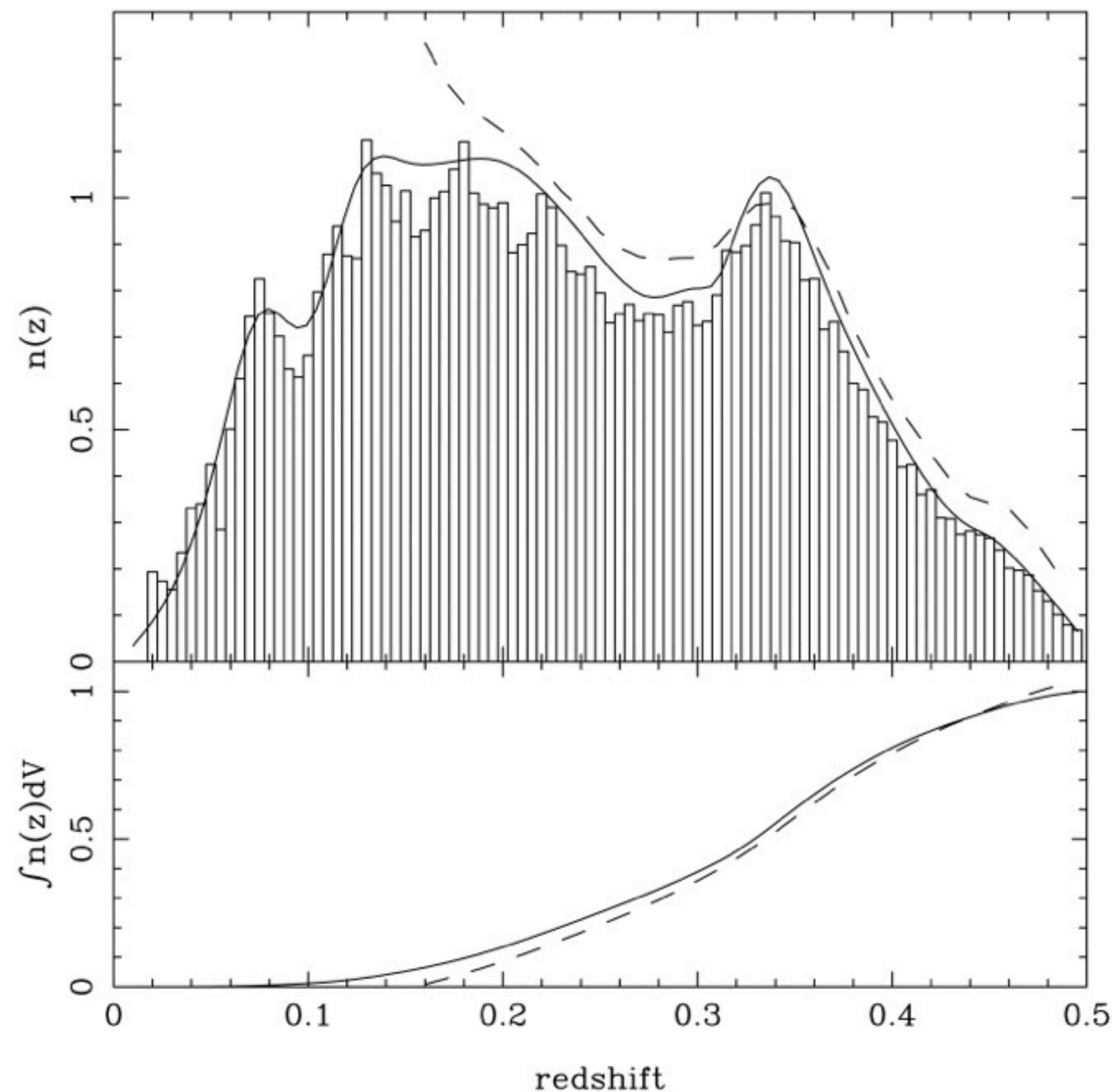
$$\begin{aligned} \delta n_A &= \delta n_B + [\bar{n}(x_A) - \bar{n}(x_B)] \\ &= \delta n_B + \bar{n}'(x_B) \underline{(\tau_A - \tau_B)} \end{aligned}$$

- Therefore, it is important to know *in which coordinate frame we observe background density!*

# Two central questions

- In which frame we measure the background number density?
- In which frame is the galaxy bias linear in matter density?

# Observed mean galaxy density



SDSS DR7 Reid et al. (2010)

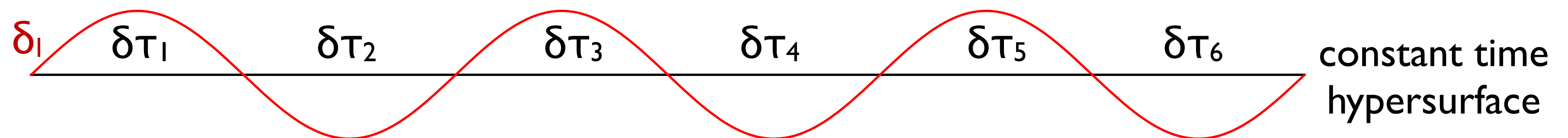
- In galaxy surveys, we estimate the background number density by angular averaging samples in a redshift bin.
- That is, we observe the galaxy density contrast reference to the **constant-observed-redshift** slicing (we call **uniform-redshift (uz) gauge**).

# Linear bias, reconsidered

- In peak-background split, we get bias from (e.g. Fabian's talk)

$$\delta_g(M, \tau) = \frac{\bar{n}(M, \delta_c - \delta_l, \tau)}{\bar{n}(M, \delta_c, \tau)} - 1 \simeq - \frac{\partial \ln \bar{n}(M, \delta_c, \tau)}{\partial \delta_c} \delta_l$$

- Hidden assumption:  $\tau$ , or  $\sigma_R(\tau)$ , is constant in space.



- Therefore, bias is linear iff the constant time hyper-surface shares same  $\sigma_R$  (for any given scale  $R$ ), or evolutionary stage, or conformal time!  $\rightarrow$  Synchronous comoving (sc) gauge!

For other argument why we need to do it in (sc) gauge, see, e.g. Wands & Slosar (2009), Bartolo et al. (2010).

# Adding up two answers

- $\delta\tau$  = time difference between (sc) and (uz) gauge

$$\delta z^{(\text{uz})} = 0 = \delta z^{(\text{sc})} + aH\delta\tau$$

- With the same time change, the density contrast of galaxies are transformed as

$$\delta_g^{(\text{uz})} = \delta_g^{(\text{sc})} - \frac{\partial \ln \bar{n}}{\partial \tau} \delta\tau = b\delta_m^{(\text{sc})} + \frac{1}{aH} \frac{\partial \ln \bar{n}}{\partial \tau} \delta z^{(\text{sc})}$$

galaxy density  
contrast in uniform  
redshift gauge

Variables in synchronous comoving gauge  
(where bias is linear in matter density)

spatial metric in (sc) gauge  $\longrightarrow g_{ij} = a^2(\tau) \left[ (1 + 2D)\delta_{ij} + 2 \left( \partial_i \partial_j - \frac{1}{3} \delta_{ij} \partial^2 \right) E \right]$

# One more step

- Assuming the universal mass function ( $f = d \ln D / d \ln a$ )

$$\frac{\partial \ln \bar{n}}{\partial \tau} = \frac{\partial \ln \bar{n}}{\partial \sigma_R} \frac{\partial \sigma_R}{\partial \tau} = a H \delta_c f (b - 1)$$

- redshift perturbation in synchronous comoving gauge

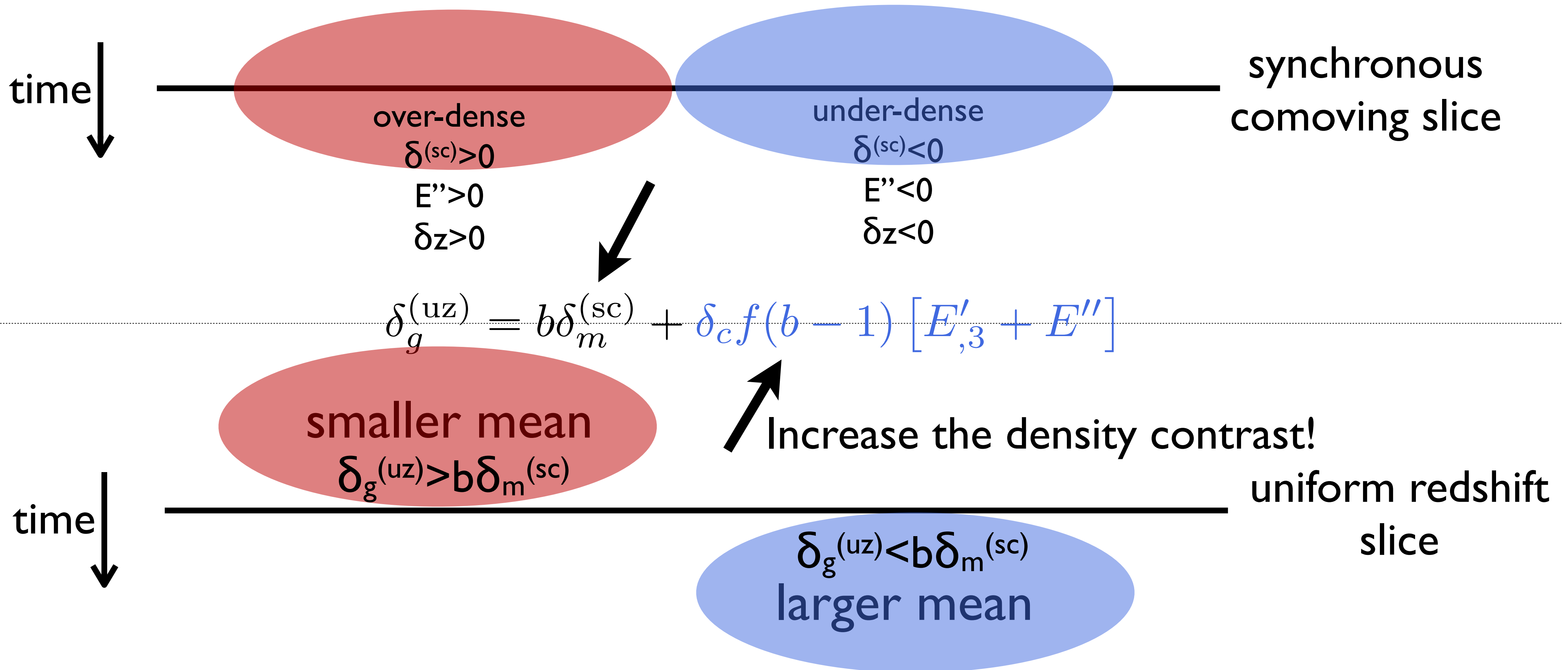
$$\delta z^{(sc)} = E'_{,3} + E'' + [\text{ISW}]$$

- Final formula for  $\delta_g^{(uz)}$  :

$$\delta_g^{(uz)} = b \delta_m^{(sc)} + \delta_c f (b - 1) [E'_{,3} + E'']$$



# Meaning of this equation (Einstein de-Sitter Universe)



# There are more GR effects!

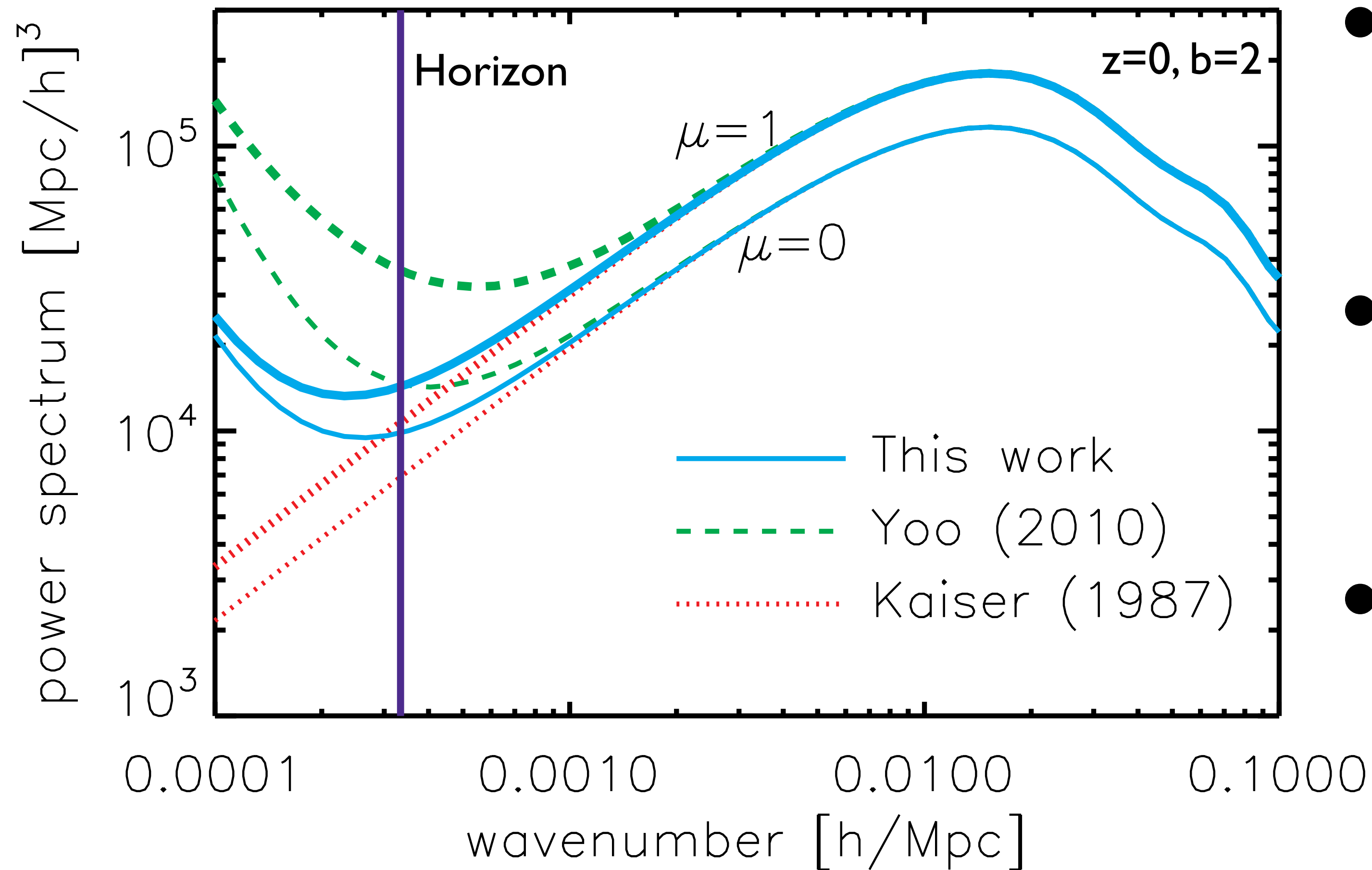
- Effect from volume, magnification in general relativity
  - Yoo et al. (2009), Yoo (2010), and his talk (next)!
- Including all effects + new bias in this work, we have

$$\begin{aligned}
 \delta_g^{\text{obs}} = & \underbrace{(b + f\mu^2)}_{\text{Kaiser (1987)}} \delta_m^{(sc)} + \underbrace{i\mu a H f [\delta_c f (b - 1) - \mathcal{C}]}_{\text{direction dep. time shift (E',3)}} \frac{\delta_m^{(sc)}}{k} \\
 & + \underbrace{\frac{3}{2} a^2 H^2 \Omega_m}_{\text{direction indep. time shift (E'')}} \left[ \delta_c f (b - 1) \left( 1 - \frac{2f}{3\Omega_m} \right) + 2 - (f - \mathcal{C}) \right] \frac{\delta_m^{(sc)}}{k^2}
 \end{aligned}$$

additional effects

$$\mathcal{C} = \frac{3}{2} \Omega_m + (5p - 2) \left( 1 - \frac{1}{aH\chi} \right) \quad \text{dn/dL} \propto \text{L}^{-(5p/2+1)}$$

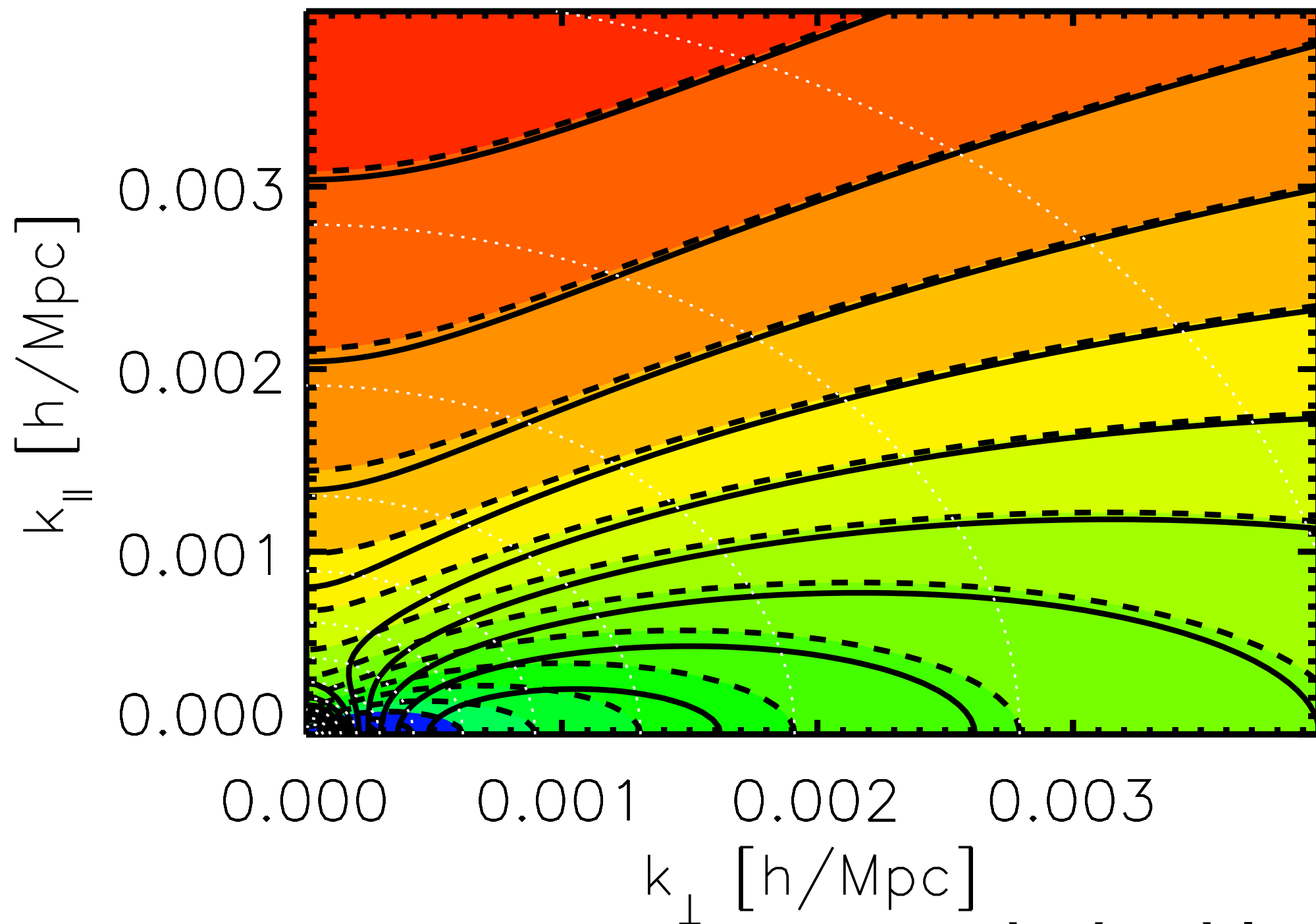
# Comp. with previous works



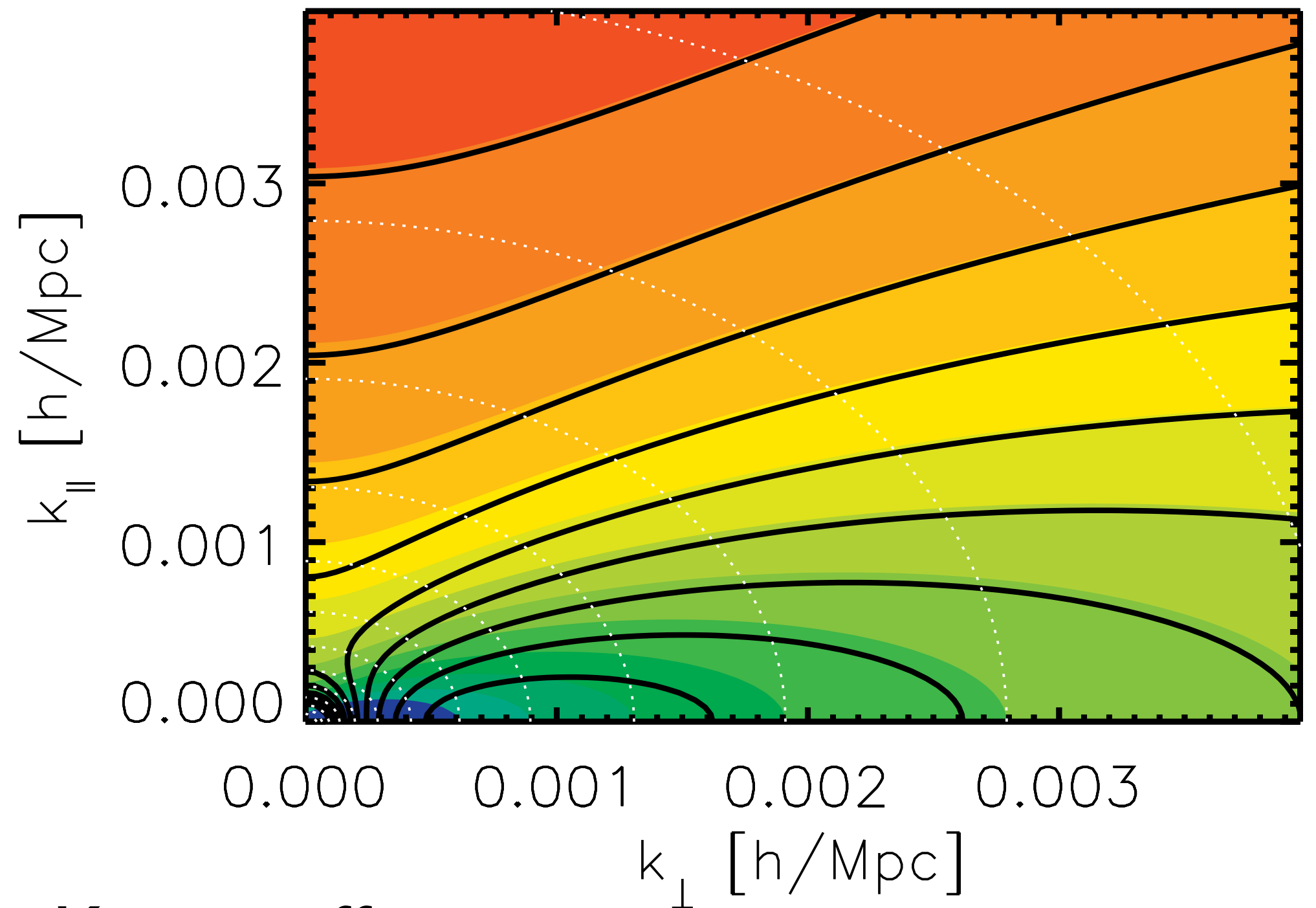
- Red = linear bias with linear redshift space distortion
- Green = linear bias in the uniform redshift gauge
- Blue = linear bias in the synchronous comoving gauge

# galaxy power spectrum (2D)

Gaussian 2D Pk with  
GR correction

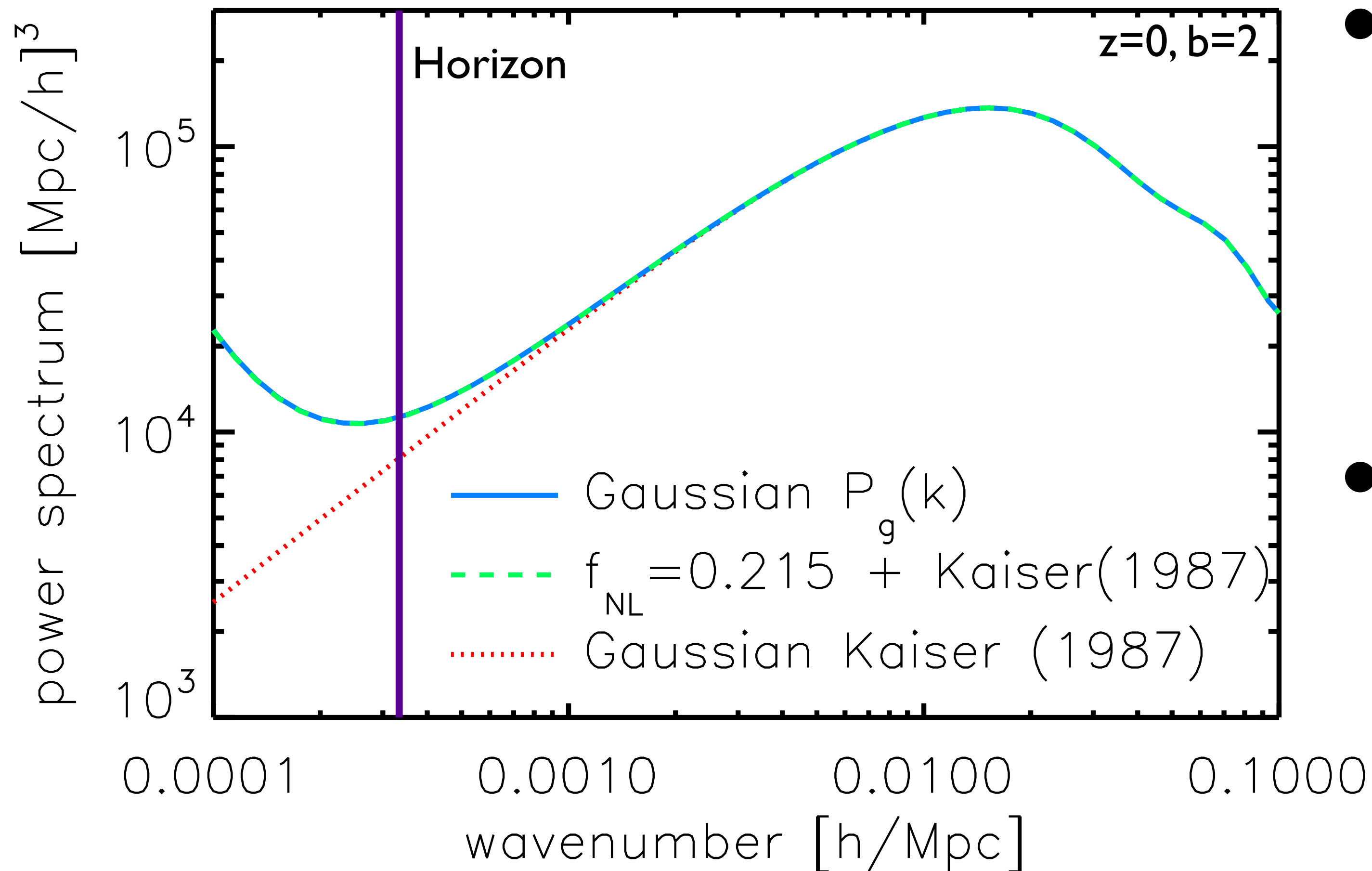


non-Gaussian 2D Pk  
with  $f_{\text{NL}}=0.215$



dashed line = Kaiser effect

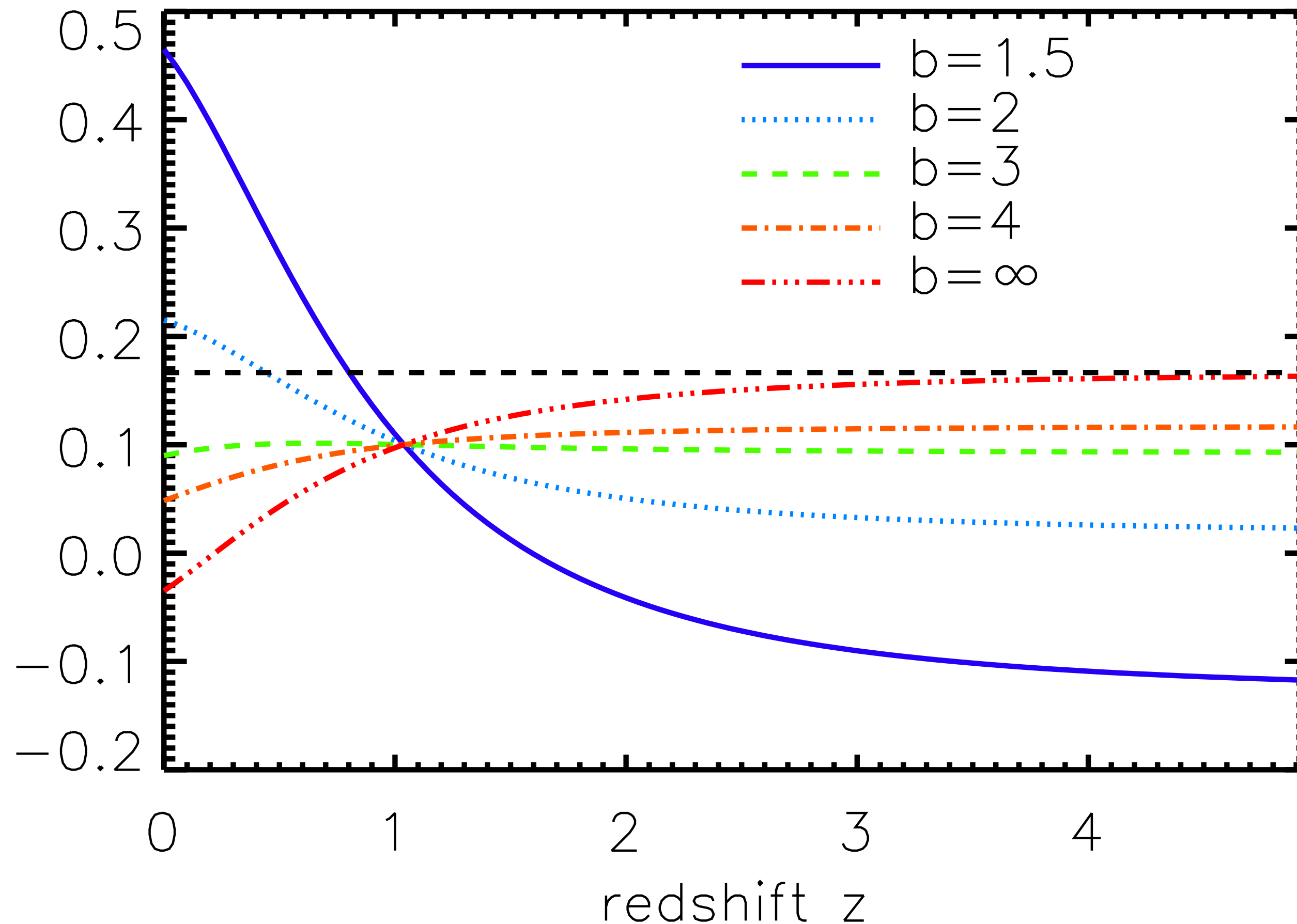
# galaxy power spectrum (1D)



- Near Horizon scales, the GR effect is dominant over the linear galaxy power spectrum
- Note: Most of deviation from the volume effect!

# Effective local $f_{\text{NL}}$

$$f_{\text{NL}}^{\text{eff}} = \frac{1}{2}g(a) \left[ f \left( 1 - \frac{2f}{3\Omega_m} \right) + \frac{2 - (f + 3\Omega_m/2)}{\delta_c(b - 1)} \right]$$



- Effective local  $f_{\text{NL}}$  varies with redshift and bias.
- Approaching  $1/6$  for high- $z$ , high  $b$
- Note: assumed  $p=0.4$

# Conclusion

- Q1: In which frame we measure the background number density?
  - [A1] Uniform redshift gauge
- Q2: In which frame is the galaxy bias linear in matter density?
  - [A2] Synchronous comoving gauge
- Combining two, we calculate the bias relation on horizon scales in synchronous comoving gauge.
- This leads  $-0.1 < f_{\text{NL}}^{(\text{eff})} < 0.5$ , when  $p=0.4$ ,  $b > 1.5$ .